RBE 3005
วิศวกรรมหุ่นยนต์ (Robotics Engineering)

สาขาวิศวกรรมหุ่นยนต์

คณะวิศวกรรมศาสตร์และเทคโนโลยีอุตสาหกรรม

มหาวิทยาลัยราชภัฏสวนสุนันทา

บทที่ 3 จลนศาสตร์ข้างหน้าและผกผัน
Forward Kinematics and Inverse Kinematics

- สายโซ่จลนศาสตร์ (kinematic chain)
- จลนศาสตร์ข้างหน้าโดยวิธีการของเดนาวิท-ฮาร์เทนบอร์ก (Denavit-Hartenberg)

Brachiation robot, Nagoya University

The brachiation robot is a mobile robot which dynamically moves from branch to branch like a long-armed ape, swinging its body like a pendulum. This video shows a brachiation robot called Brachiator II. Brachiator II is the two-link robot which takes the form of a double pendulum. This video is presented through the courtesy of a co-author Prof. Toshio Fukuda (Meijo University, Nagoya University, and Beijing Institute of Technology).

https://youtu.be/wZi4ySqSBUg?feature=shared

review

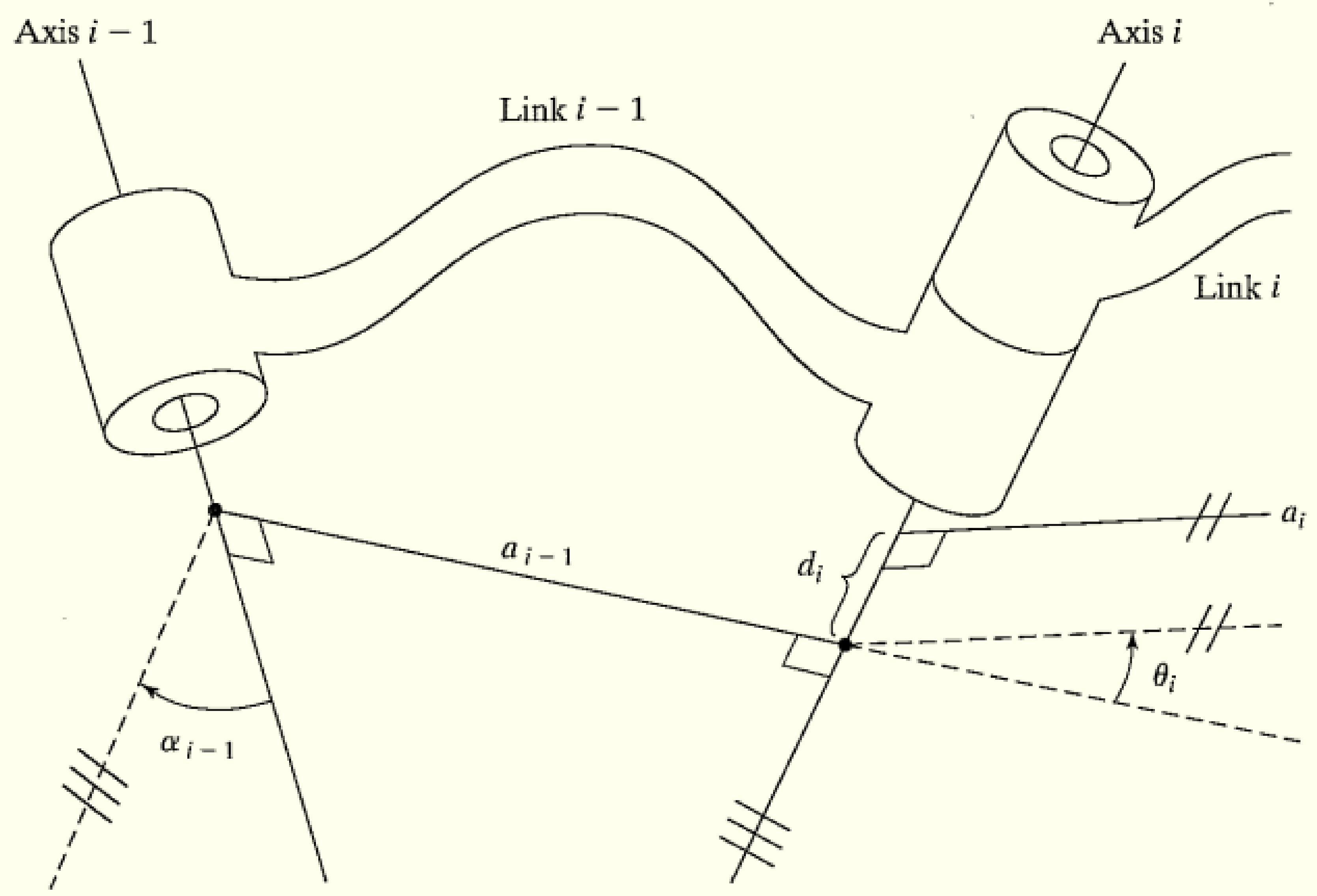
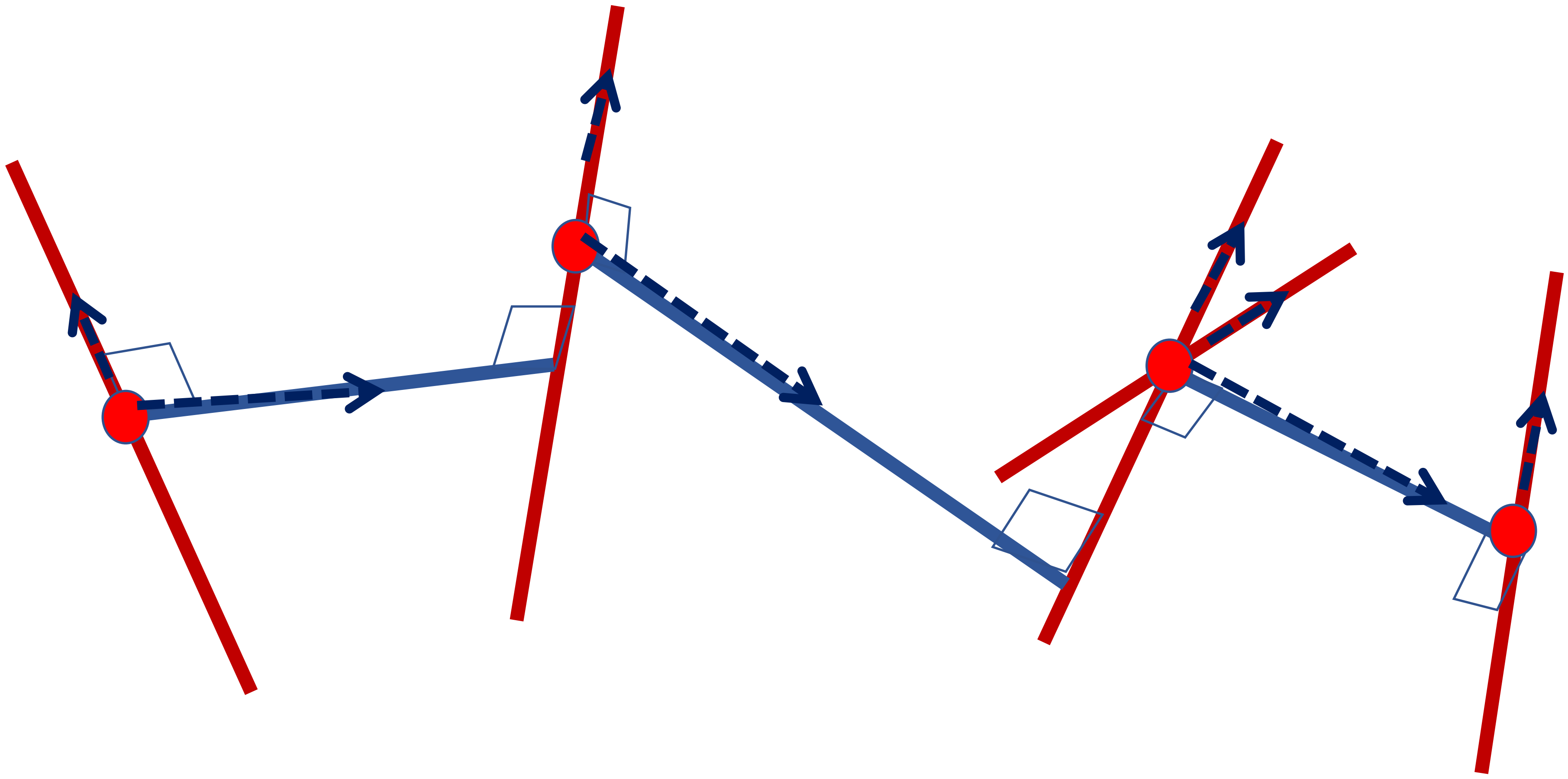
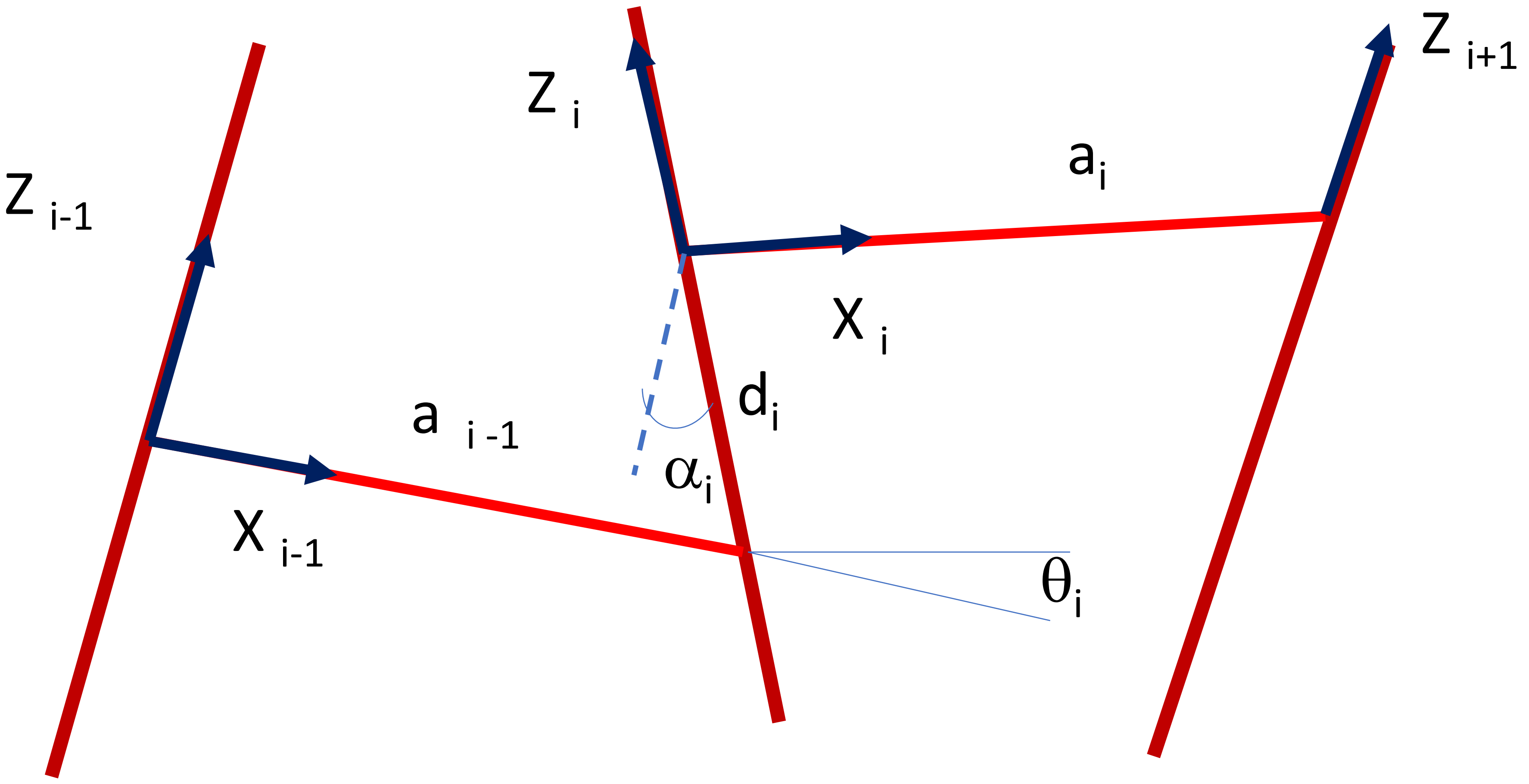


FIGURE 3.4: The link offset, d, and the joint angle, θ, are two parameters that may be used to describe the nature of the connection between neighboring links.

Summary – Frame Attachment



1. Normals
2. Origins
3. Z-axes
4. X-axes



a_i : distance (Z_i, Z_{i+1}) along X_i

α_i : angle (Z_i, Z_{i+1}) along X_i

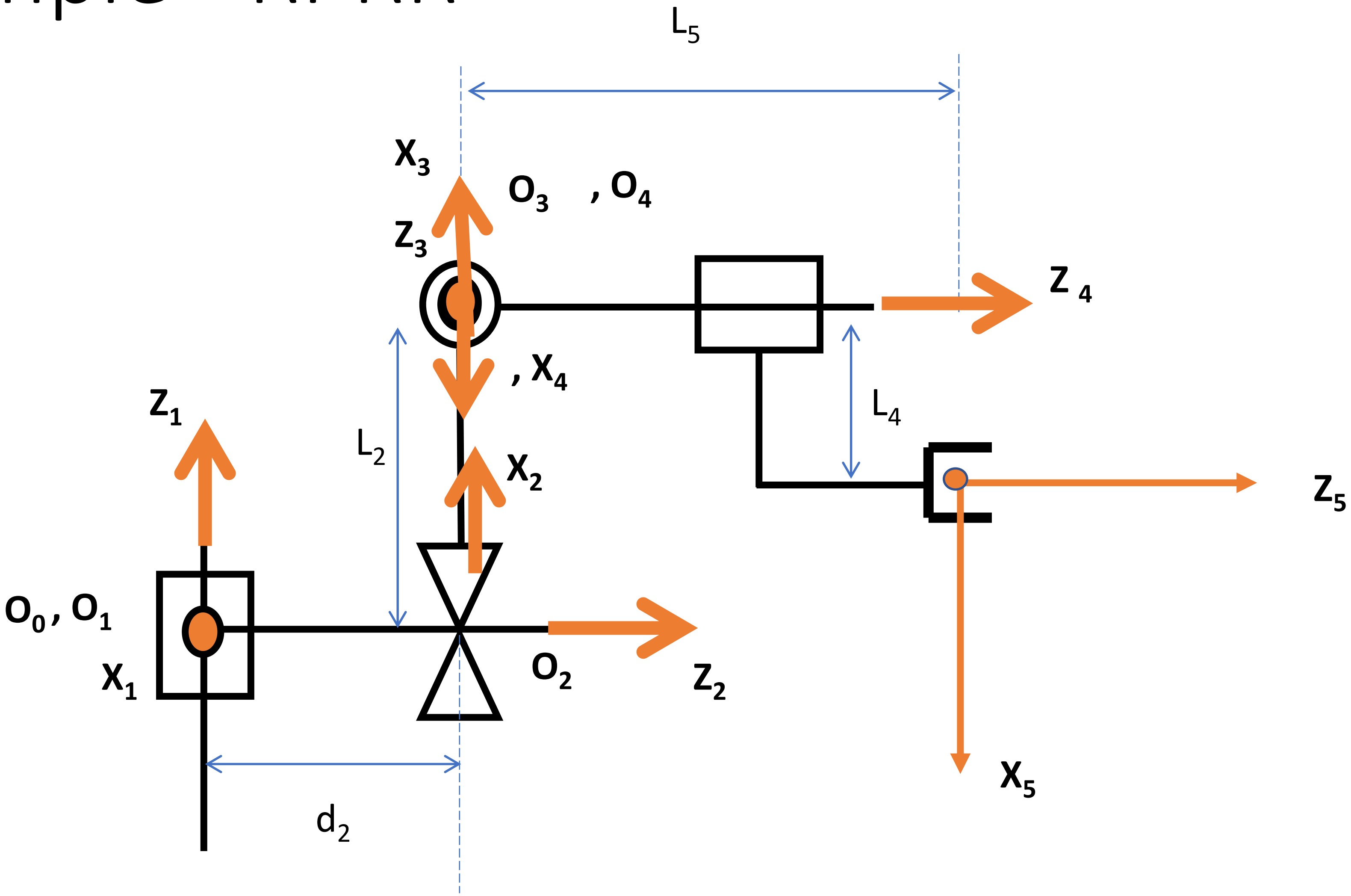
d_i : distance (X_{i-1}, X_i) along Z_i

θ_i : angle (X_{I-1}, X_i) along Z_i

D-H parameter and Frame attachment

- Procedure
  - Normal
  - Origin
  - Z-axis
  - X-axis
  - Define D-H parameter written in table
  - Transformation Matrix between end effector and base frame {0}

Example - RPRR



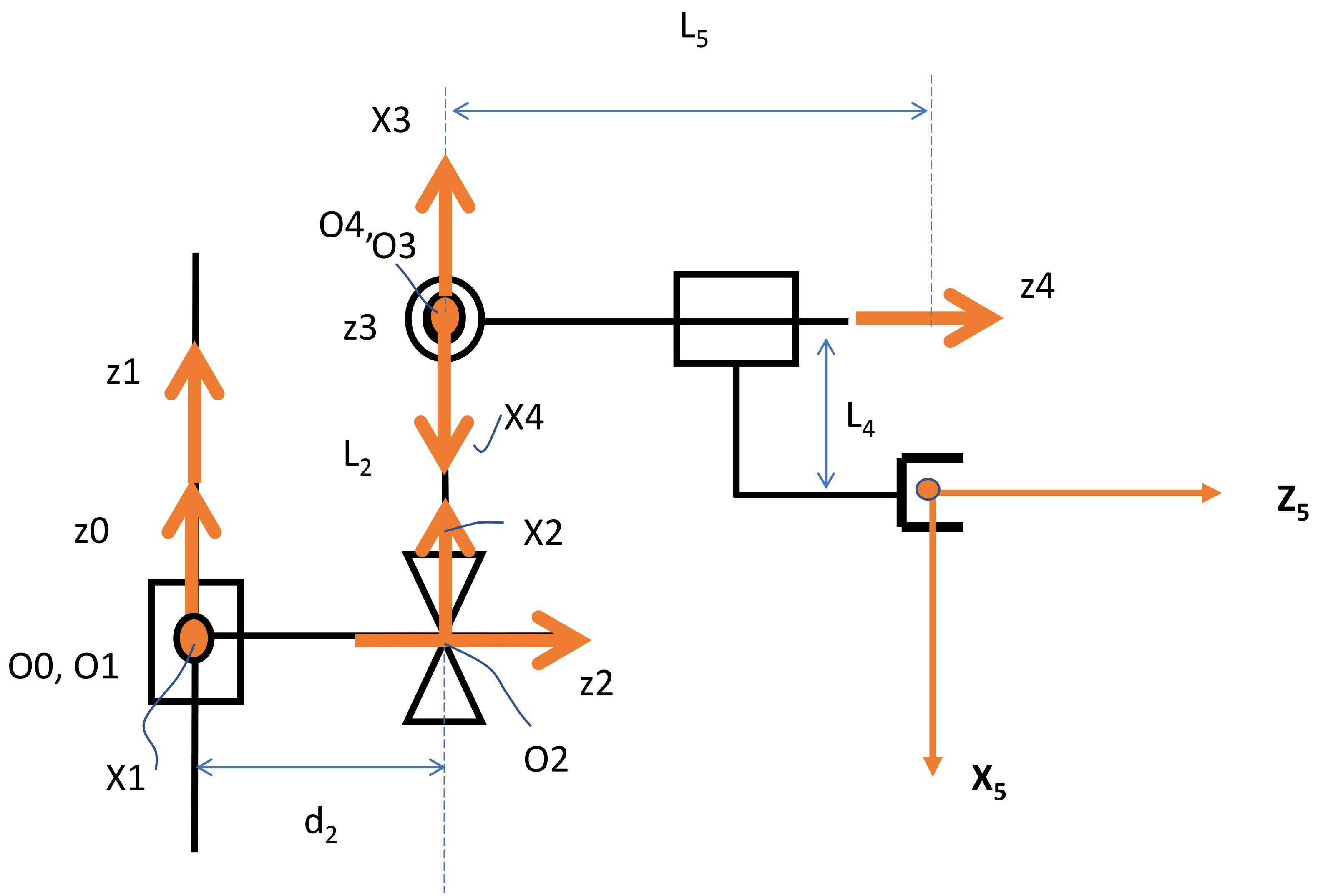
i	**a_{i-1}**	**α_{i-1}**	**d_i**	**θ_i**
1	0	0	0	θ_1
2	0	-90°	d_2	-90°
3	L_2	-90°	0	θ_3
4	0	90°	L_5	θ_4
5	L_4	0°	0	0

a_i : distance (Z_i , Z_{i+1}) along X_i

α_i : angle (Z_i , Z_{i+1}) about X_i

d_i : distance (X_{i-1} , X_i) along Z_i

θ_i : angle (X_{I-1} , X_i) about Z_i


L5
X3
O4, O3
z4
z3
z1
X4
L2
L4
z0
X2
Z5
O0, O1
z2
O2
X1
X5
d2

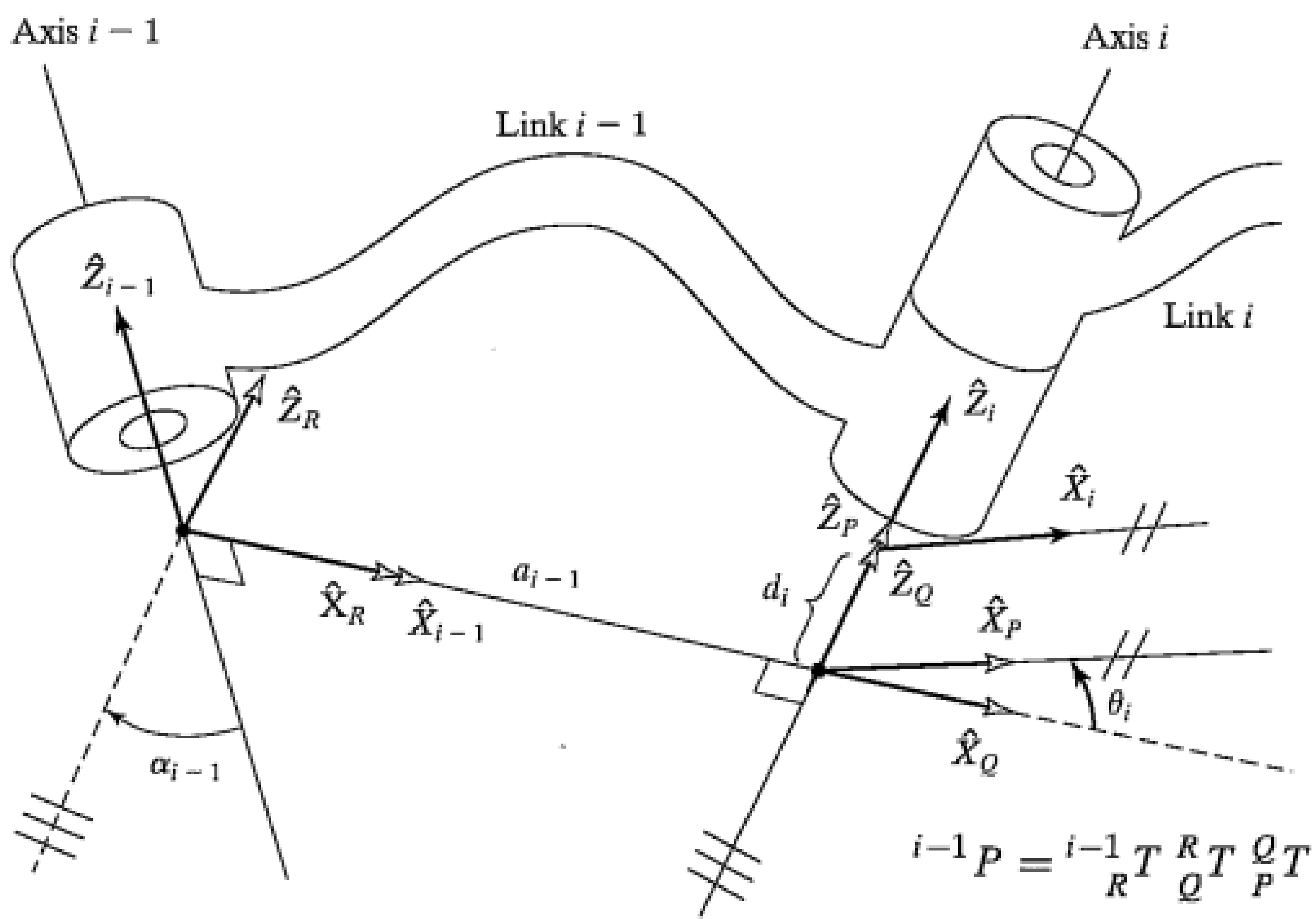
i	**a_{i-1}**	**α_{i-1}**	**d_i**	**θ_i**
1	0	0	0	θ_1
2	0	-90	d2	-90
3	L2	-90	0	θ_3
4	0	90	L_5	θ_4
5	L4	0	0	0

a_i : distance (Z_i , Z_{i+1}) along X_i

α_i : angle (Z_i , Z_{i+1}) about X_i

d_i : distance (X_{i-1} , X_i) along Z_i

θ_i : angle (X_{I-1} , X_i) about Z_i



$$ {}^{i-1}P = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T \; {}^{i}P, \tag{3.1} $$

FIGURE 3.15: Location of intermediate fram

$$ {}^{i-1}P = {}^{i-1}_{i}T \; {}^{i}P, \tag{3.2} $$

$$ {}^{i-1}_{i}T = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T. \tag{3.3} $$

Considering each of these transformations, we see that (3.3) may be written

$$ {}^{i-1}_{i}T = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i), \tag{3.4} $$

or

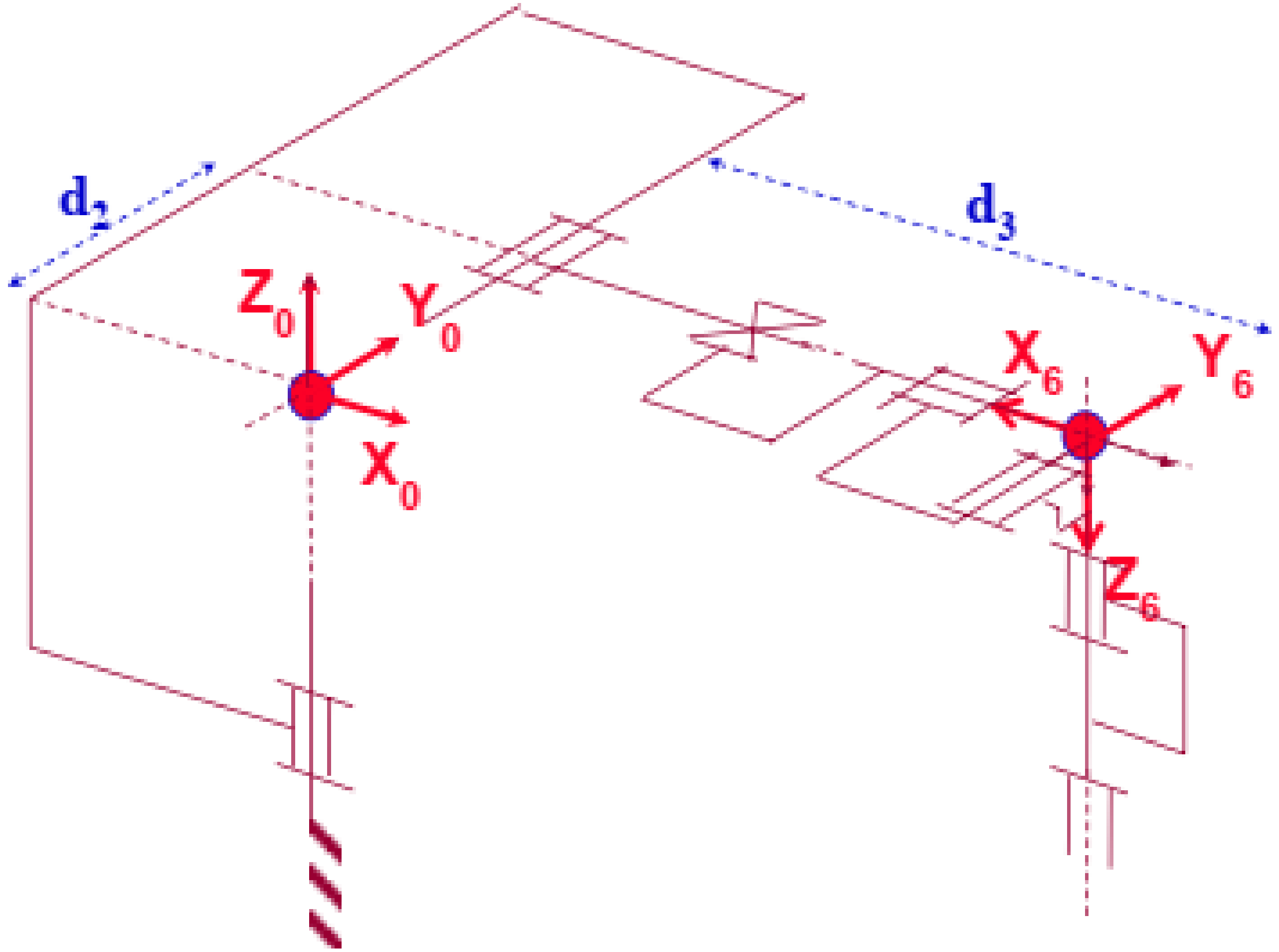
$$ {}^{i-1}_{i}T = \mathrm{Screw}_X(a_{i-1}, \alpha_{i-1})\ \mathrm{Screw}_Z(d_i, \theta_i), \tag{3.5} $$

where the notation $\mathrm{Screw}_Q(r, \phi)$ stands for the combination of a translation along an axis $\hat{Q}$ by a distance r and a rotation about the same axis by an angle ϕ. Multiplying out (3.4), we obtain the general form of ${}^{i-1}_{i}T$:

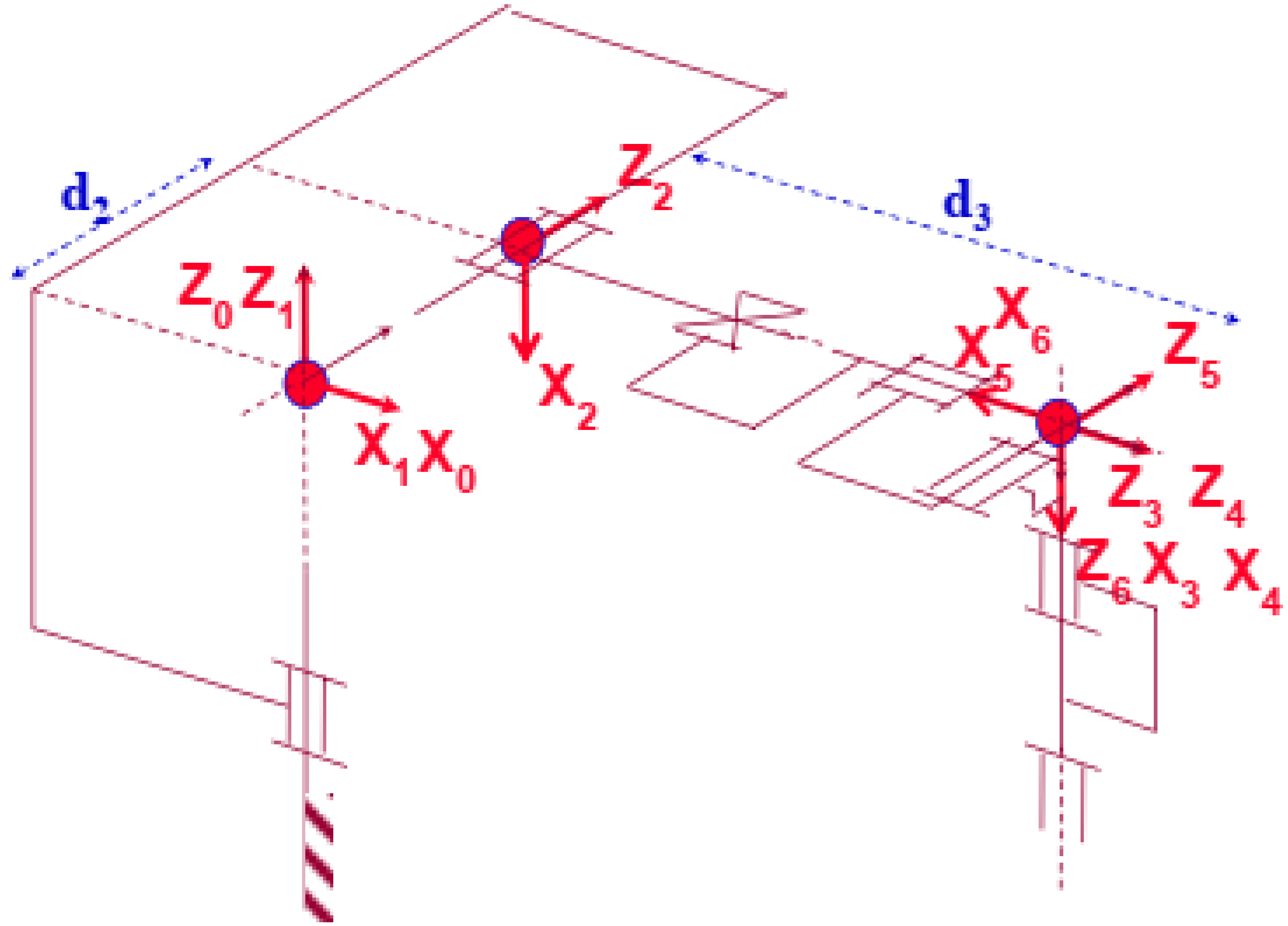
$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.6} $$

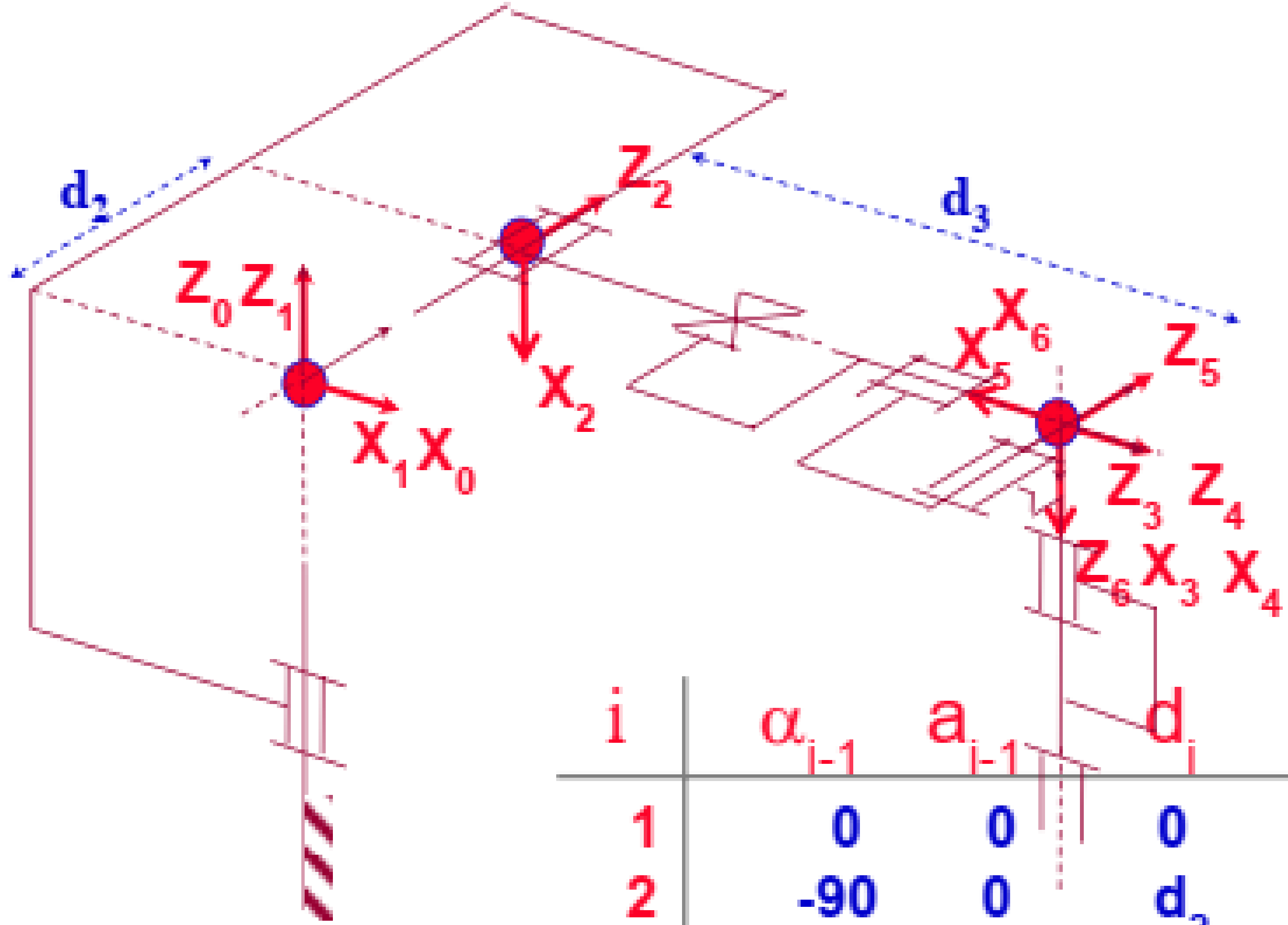
Stanford Scheinman Arm

Stanford Scheinman Arm



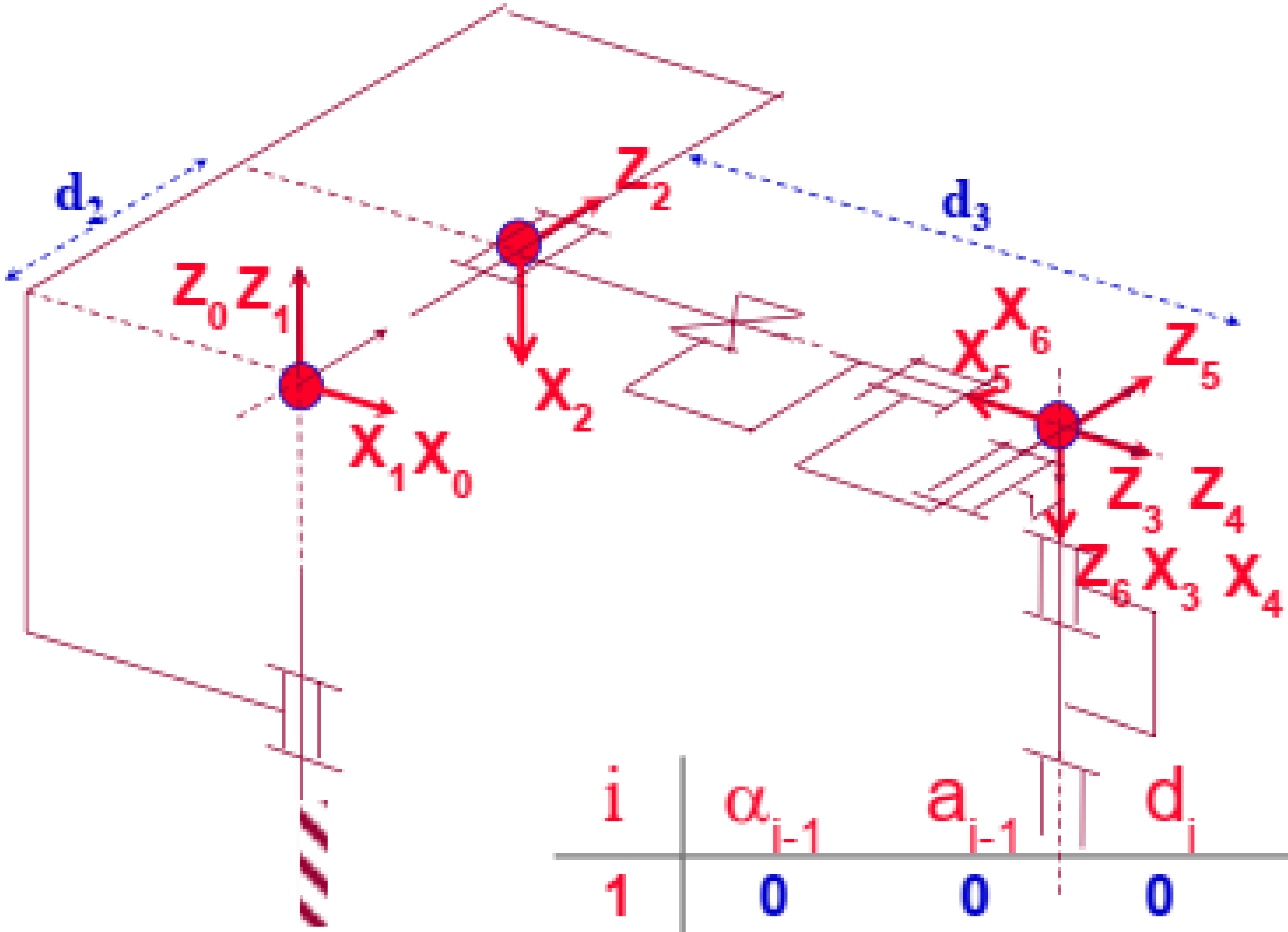
Stanford Scheinman Arm





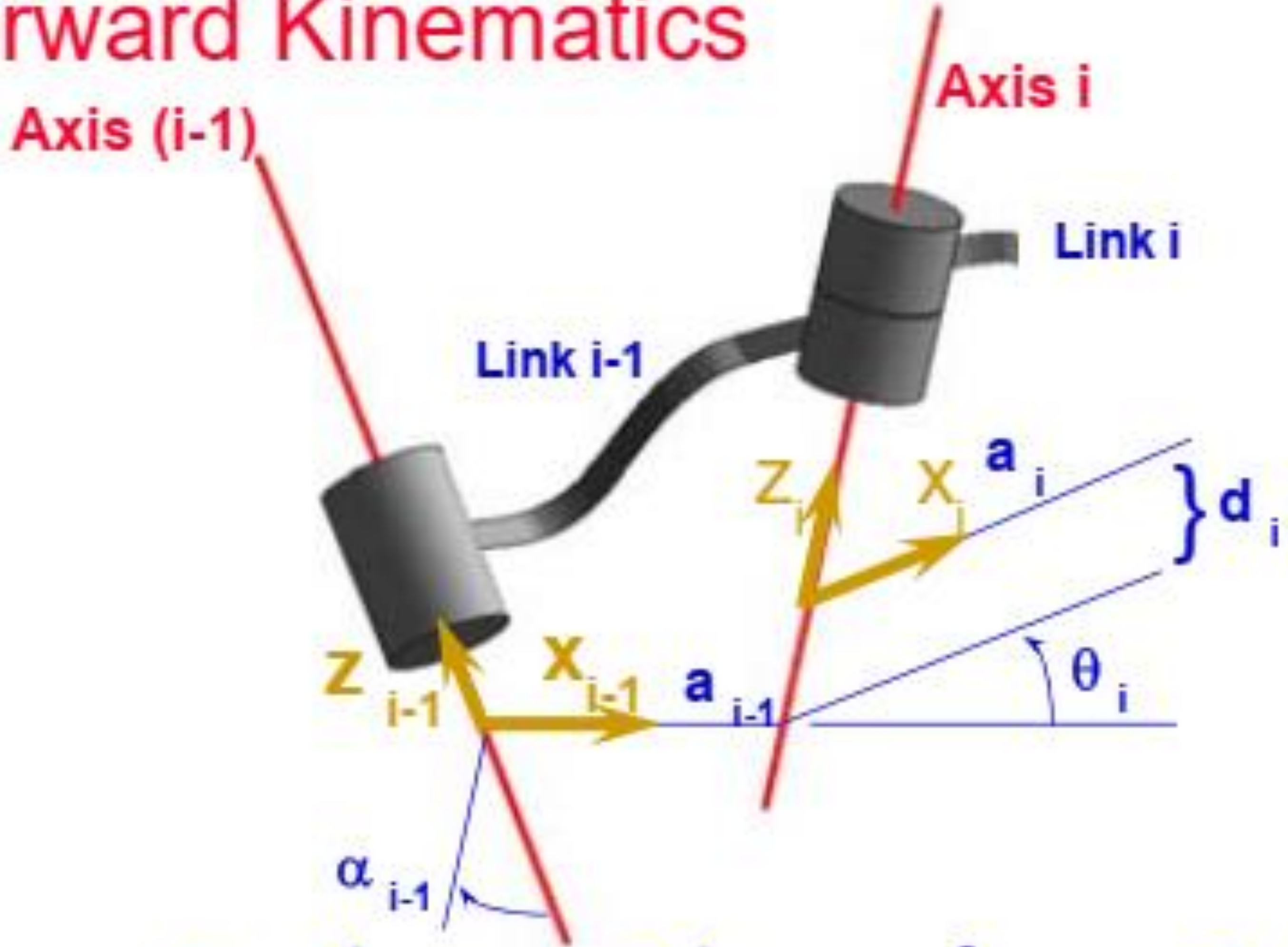
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	d_3	0
4	0	0	0	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6

a_i : distance (z_i , z_{i+1}) along x_i
α_i : angle (z_i , z_{i+1}) about x_i
d_i : distance (x_{i-1} , x_i) along z_i
θ_i : angle (x_{i-1} , x_i) about z_i



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	d_3	0
4	0	0	0	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6

Forward Kinematics



$$
{}^{i-1}_{1}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i \, c\alpha_{i-1} & c\theta_i \, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} \, d_i \\ s\theta_i \, s\alpha_{i-1} & c\theta_i \, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} \, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Stanford Scheinman Arm

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	d_3	0
4	0	0	0	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6

$$\begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

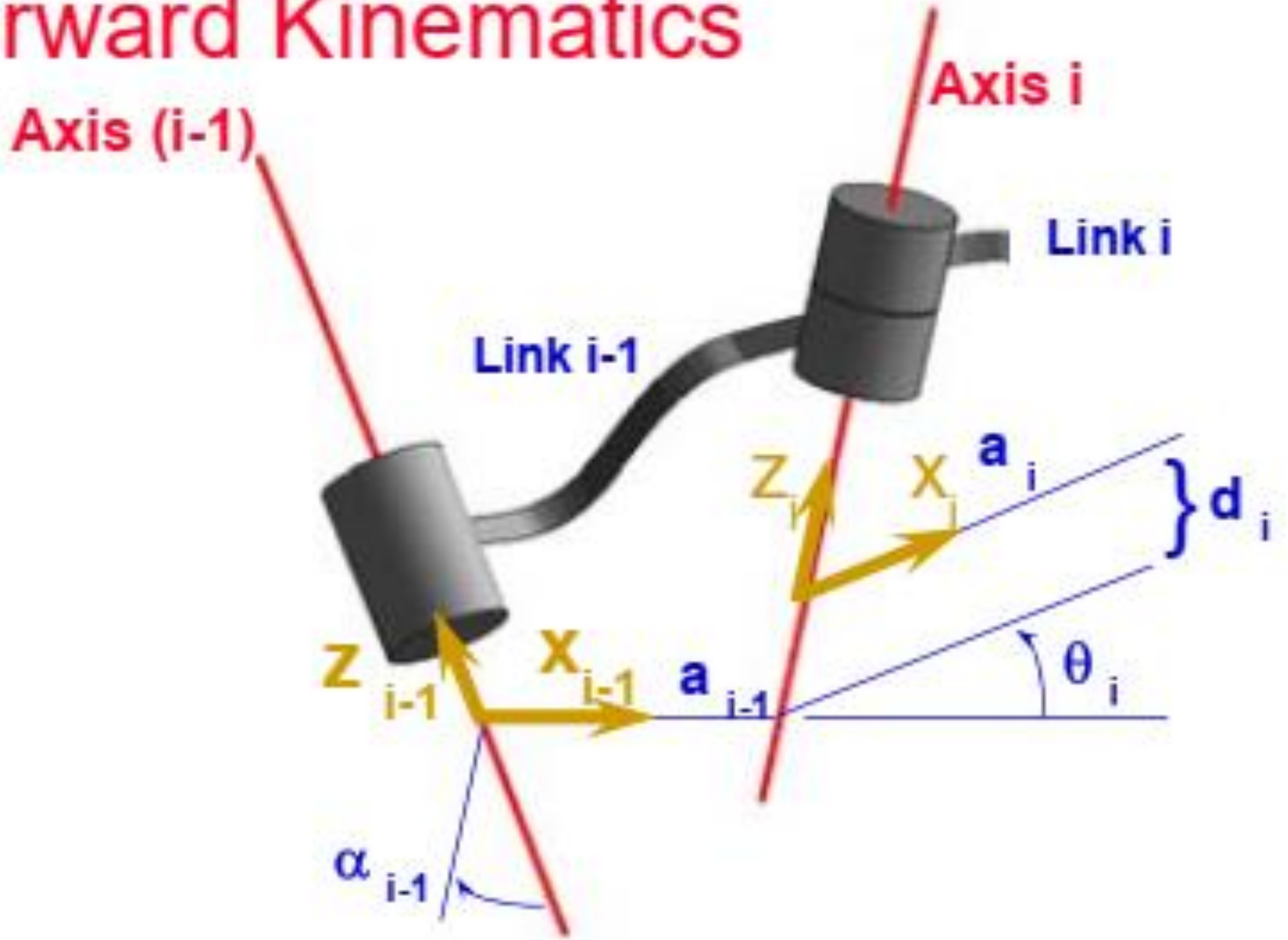
$${}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{3}_{4}T = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}_{5}T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}_{6}T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics



Forward Kinematics: $^{0}_{N}T = {}^{0}_{1}T \; {}^{1}_{2}T \; \ldots \; {}^{N-1}_{N}T$

$$
{}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{2}T = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & -s_1 & -s_1 d_2 \\ s_1 c_2 & -s_1 s_2 & c_1 & c_1 d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

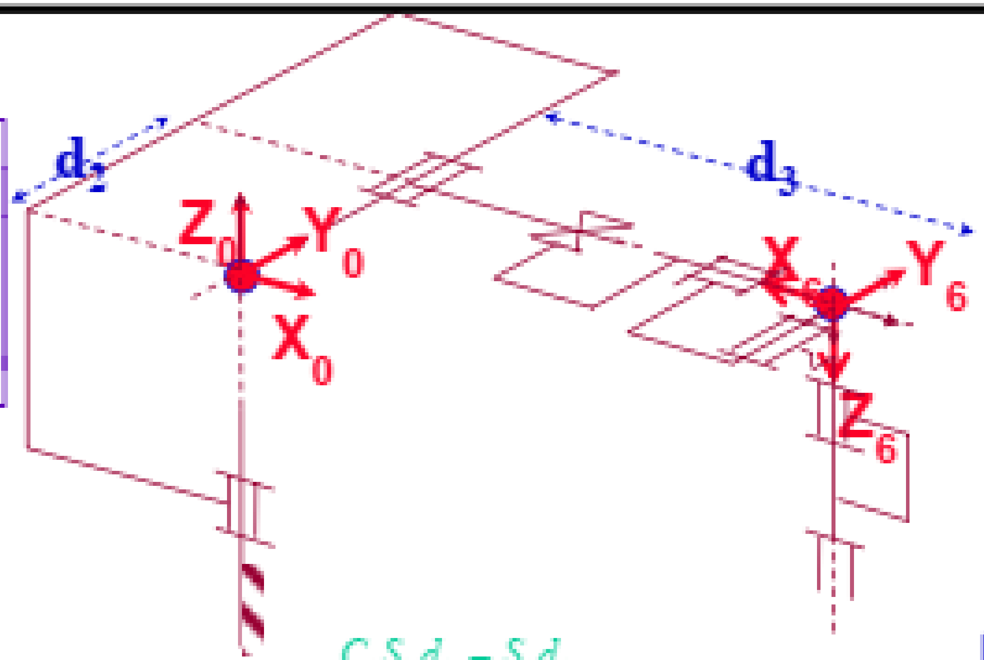
$$
{}^{0}_{3}T = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & c_1 d_3 s_2 - s_1 d_2 \\ s_1 c_2 & c_1 & s_1 s_2 & s_1 d_3 s_2 + c_1 d_2 \\ -s_2 & 0 & c_2 & d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{4}T = \begin{bmatrix} c_1c_2c_4 - s_1s_4 & -c_1c_2s_4 - s_1c_4 & c_1s_2 & c_1d_3s_2 - s_1d_2 \\ s_1c_2c_4 + c_1s_4 & -s_1c_2s_4 + c_1c_4 & s_1s_2 & s_1d_3s_2 + c_1d_2 \\ -s_2c_4 & s_2s_4 & c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{5}T = \begin{bmatrix} X & X & -c_1c_2s_4 - s_1c_4 & c_1d_3s_2 - s_1d_2 \\ X & X & -s_1c_2s_4 + c_1c_4 & s_1d_3s_2 + c_1d_2 \\ X & X & s_2s_4 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{6}T = \begin{bmatrix} X & X & c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2s_5 & c_1d_3s_2 - s_1d_2 \\ X & X & s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 & s_1d_3s_2 + c_1d_2 \\ X & X & -s_2c_4s_5 + c_5c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{6}T=\begin{bmatrix} X & X & c_1c_2c_4s_5-s_1s_4s_5+c_1s_2s_5 & c_1d_3s_2-s_1d_2 \\ X & X & s_1c_2c_4s_5+c_1s_4s_5+s_1s_2c_5 & s_1d_3s_2+c_1d_2 \\ X & X & -s_2c_4s_5+c_5c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



$$
x=\begin{pmatrix} x_P \\ r_1 \\ r_2 \\ r_3 \end{pmatrix}=\left|\begin{matrix} C_1S_2d_3-S_1d_2 \\ S_1S_2d_3+C_1d_2 \\ C_2d_3 \\ C_1[C_2(C_4C_5C_6-S_4S_6)-S_2S_5C_6]-S_1(S_4C_5C_6+C_4S_6) \\ S_1[C_2(C_4C_5C_6-S_4S_6)-S_2S_5C_6]+C_1(S_4C_5C_6+C_4S_6) \\ -S_2(C_4C_5C_6-S_4S_6)-C_2S_5C_6 \\ C_1[-C_2(C_4C_5S_6+S_4C_6)+S_2S_5S_6]-S_1(-S_4C_5S_6+C_4C_6) \\ S_1[-C_2(C_4C_5S_6+S_4C_6)+S_2S_5S_6]+C_1(-S_4C_5S_6+C_4C_6) \\ S_2(C_4C_5S_6+S_4C_6)+C_2S_5S_6 \\ C_1(C_2C_4S_5+S_2C_5)-S_1S_4S_5 \\ S_1(C_2C_4S_5+S_2C_5)+C_1S_4S_5 \\ -S_2C_4S_5+C_2C_5 \end{matrix}\right|
$$